Nonlinear Frequency Domain Filter for the Quasi-Periodic Noise Removal

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ABSTRACT

This paper is dedicated to the periodic noise filtering. The goal is to introduce two novel filters that act in the Fourier amplitude spectrum, to remove the periodic and quasi-periodic noise. The idea of the filters is an automatic peaks detection in the amplitude spectum and their further elimination using the median calculated from the filter window. Both filters show excellent results not only for the images whose dimensions are equal to power of two (which is the limitation of Cooley-Tukey's FFT algorithm), but for the images of arbitrary sizes.

1 INTRODUCTION

The problem of periodic and quasi-periodic pattern removal from photographs is an important problem. There are a lot of sources of this periodic noise. For example, the resolution of the scanner used to scan the image affects the high frequency noise pattern in the acquired image and can produce moire patterns. The same problem is usual for the images obtained from single-chip video cameras. One can see the examples of periodic noise and their amplitude spectrum on Fig. 1. The goal of this paper is to introduce two new algorithms for periodic and quasi-periodic noise filtering, which will be reduced to the effective semi-automatic noise detection and nonlinear correction of the image's amplitude spectrum.

A usual approach for quasi-periodic noise filtering is the frequency domain processing. It is a much better solution than any kind of spatial domain blurring, which often is unable to reduce a periodic structure, but at the same time it reduces the edge sharpness significantly. Any periodic and quasi-periodic noise leads to the appearance of peaks in the image's amplitude spectrum (see for example, [1]). This is true for the Fourier, Cosine and Hadamard-Walsh spectra, for example. In the case of periodic nose, these peaks are even visible to an eye. The peaks corresponding to the quasi-periodic noise usually are hidden and sometimes it is difficult to detect them without some kind of numeric analysis.

To solve the periodic noise elimination problem, these peaks have to be found and removed. All the solutions are concentrated on these two key points. Until now we had seen only such solutions as the Wiener filter[1] and the spectrum domain peak elimination using the mask (created by the thresholding of the amplitude spectrum), described, for example, in [2], [3]. The Wiener filter is good in general, but to apply this filter, a proper and precise noise model must be built, which is a complicate task. Furthermore, the Wiener filter is computationally slow. The second approach, which is reduced to the amplitude spectrum's thresholding, can't be considered as an optimal solution. This approach is suitable only for the truly periodic noise, which implies high peaks in the image's amplitude spectrum. But the quasi-periodic noise doesn't produce the strong peaks that can be detected using the thresholding (for an example see Section 2 and Fig. 2).

A good idea is to find some algorithm for the automatic peaks' detection and elimination. This was the goal of our work. We are trying to present here a general solution, which will cover both periodic and quasi-periodic noise cases. The proposed algorithm consists of the automatic detector of the periodic and quasi-periodic noise in the frequency domain and peak filter.

A very strong criterion for the automatic detection even of the small peaks in the amplitude spectrum is developed. After the peak detection the median filter or Gaussian median filter, which are introduced in this paper, can be used for their elimination.

The following aspects of the problem are considered in the paper. To detect peaks prior to filtering (see section 2) we calculate a median of the local window in the amplitude spectrum. Then we are analyzing a ratio of the coefficient of interest to the median. This ratio is compared to the threshold. If the threshold is exceeded then either the coefficient could be replaced by the median (see section 3) and actually this is the median filtering. Another filter is obtained, when the whole local window is multiplied by a concave (Gaussian-like) surface (with values varying from 0 to 1) to efficiently remove wide peaks (see section 4). This is the filter, which we call the Gaussian median filter. It is especially effective

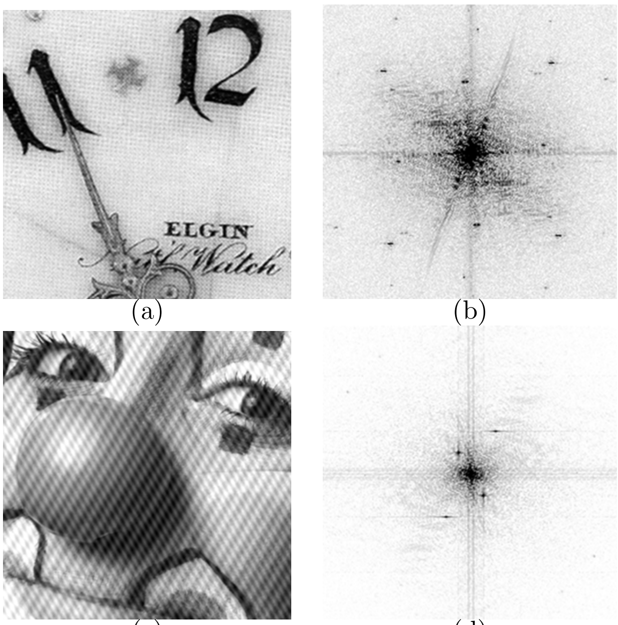

Figure 1: Examples of the quasi-periodic noise and the corresponding Fourier amplitude spectra . It is clearly visible that the periodic structires are reflectinf as a series of peaks in the amplitude spectrum. (a) and (b) Image "Clock" and its spectrum; (c) and (d) Image "Clown" and its spectrum.

for the complicate quasi-periodic noise removal. The results are compared to the common spectrum thresholding technique in the Section 5 (and also in Section 2, Fig. 2).

2 PEAK DETECTOR FOR THE FOURIER AMPLITUDE SPECTRUM

As it was mentioned in the introduction, we are going to consider two filters for the periodic noise filtering. Both of them perform in the Fourier amplitude spectrum, which is described by the following formula:

$$C(k,l) = \left| \frac{1}{N_1 N_2} \sum_{m=0}^{N_1-1} \sum_{n=0}^{N_2-1} X(m,n)\, W_1^{km} W_2^{nl} \right| \quad (1)$$
$$W_1 = e^{-\frac{2\pi}{N_1} i}, W_2 = e^{-\frac{2\pi}{N_2} i},$$

where $C(k,l)$ is the resulting modulo of the spectral (k,l)-th coefficient, X is the $N_1 \times N_2$ sized image whose spectrum is calculated.

It is well known, and it can be easily seen (Fig. 1), that periodic and quasi-periodic patterns leave peaks in the amplitude spectrum. A number of these peaks is dramatically smaller in comparison to number of all the pixels or all the spectral coefficients. To perform any filtering, it is necessary to localize these peaks. We are going here to propose a simple, but an effective detector, which will solve the peaks' detection problem almost automatically.

It should be noted that we need to define a low-frequency area, where we want all the spectral coefficients to be preserved from any changes. The lowest frequency coefficients contribute a lot to the signal energy, and their change may lead to appearance of undesirable noise.

We will consider the Fourier spectrum having zero frequency coefficient in its center. Thus we can define the domain of preservation as a circle around the zero frequency coefficient $C(0,0)$. To simplify a practical implementation, it is also possible to consider this area as a rectangle with the zero frequency coefficient in its center. We will denote the domain, where all the spectral coefficients have to be preserved, by R_{LF}.

Peaks' detection algorithm, which we would like to propose here, is based on their consideration as impulses. Let us take a look at the amplitude spectra on Fig. 1bd. The bright white spots that are corresponding to the peaks look very similar to the impulses. It is well known that the median filter is the most common and usual impulsive noise filter [6]. Moreover, a median itself could be successfully used for the detection of impulses. In [5] the criterion for impulsive noise detection on the images has been considered. This criterion is based on the comparison of the current signal value to a local median value. We can put the same idea to the background of the peaks' detection in the amplitude spectrum.

The considered detector can be described in the following way. The (i,j)-th spectral coefficient C_{ij} is a peak, if the following inequality holds:

$$\frac{C_{ij}}{\underset{m\times n}{MED}(C_{ij})} \geq \theta,\ (i,j) \notin R_{LF} \quad (2)$$

where $\underset{m\times n}{MED}(C_{ij})$ is a local median in a $m\times n$ window around the (i,j)-th spectral coefficient. A threshold θ is the main parameter of the detection algorithm. It defines the strength of the detector and affects a number of the coefficients detected as peaks. Larger θ leads to more careful and light peaks' detection. In this case the number of possible misdetections is lower. At the same time it is necessary to choose in each case a relevant value of θ, which will ensure the most effective detection with a minimal number of misdetections. If θ is taken smaller than some minimum - the weird nonuniform noise starts to appear on the image together with removal of the quasi-periodic structures.

The philosophy of the detector (2) is based on the consideration that for the spectrum without the peaks a ratio $\frac{C_{ij}}{\underset{m\times n}{MED}(C_{ij})}$ is close to 1. So if some C_{ij} is unexpectedly larger than a local median value then C_{ij} is probably a peak. By the way this consideration gives a lower estimation for the value of θ: $\theta > 1$. On the other hand the choice of a proper value of θ is not a problem in each particular case. The analysis of amplitude spectrum statistics shows that the sensitivity of the detector

(2) to changes of θ is very low. It means that even in the complicate case it is very easy to find a proper value changing θ with a step equal to 0.5 or 1.

We can compare this technique to the commonly used one, which is amplitude spectrum's thresholding. If simple thresholding is taken, it is clear that for large threshold's values only highest peaks will be detected. At the same time for the small threshold values all the low frequencies will be detected as peaks and there will be no way to extract real peaks. This is due to the fact that most of the image's energy is accumulated in the low-frequency coefficients that have much higher amplitudes than medium and high frequency spectral coefficients. Let us consider a typical example (see Fig. 2). The image displayed on Fig. 2a contains diagonal lines. Thresholding, which will not destroy the image, is able to detect only those peaks that lie on the main diagonal (see Fig. 2f), while it is clearly visible that there are also peaks in high frequencies (bright spots on the Fig. 2b). A problem is that if we are going to decrease a threshold for extra peaks detection, a level of misdetection will be very high and unacceptable because the image may be more corrupted than filtered. After the thresholding a filtering mask was created (Fig. 2f) by clearing the low frequencies. This mask was used to remove the peaks. At the same time it is clear that some peaks in the high frequency domain are not detected and will not be removed. As a result, after the commonly used replacement of the detected peaks with 0 the image is not completely clear from the periodic structure. Despite this structure is very light, it is still present. It is especially clearly visible after the the edge detection. For example, after the Sobel edge detection (Fig. 2h), the leftovers of the periodic diagonal lines are clearly seen. Fig. 2e, shows a map of the peaks that were detected by the median criterion (2). Comparison with the Fig. 2b and the Fig. 2f shows that all the important peaks have been detected using the detector (2) while a standard approach failed. To summarize, we can conclude that the median peaks detector (2) is much stronger and effective than the usual thresholding. Furthermore in example presented on Fig. 2 it is hard to remove peaks manually, because there are too much of them.

At the same time it is not enough only to detect the peaks. It is necessary to eliminate them in the most effective way. From our point of view the replacement of the detected peaks by 0, which is widely used, is not a good solution because the corresponding spectral coefficients are removed in this way at all, which leads to corruption of an image, distortion of the image details. Moreover, for the complicate quasi-periodic noise it is not enough to remove the peaks themselves because often the neighboring to them spectral coefficients also must be corrected for the effective noise removal.. This is a reason to go on and to develop more efficient technique not only for the noise detection, but also for its filtering.

3 SPECTRAL MEDIAN FILTER

Spectral median filter is a usual median filter applied to the amplitude spectrum excluding a domain R_{LF} and connected to the median peak detector (2). As it was mentioned above it is possible to consider the peaks as impulses. It means that it is possible to use the median filter, which is the most common eliminator of the impulsive noise [6] to remove the peaks. To correct just those spectral coefficients, which are really peaks and to preserve all the other ones, the median filter has to be connected with the peak detector. This connection means the following. We analyze a local $m \times n$ window around each spectral coefficient. Using the detector (2) we are checking each coefficient, which does not belong to the domain R_{LF}, if it is a peak or not. If the detector gives a positive response, this spectral coefficient (amplitude spectrum's coefficient) is replaced by the local median value calculated for the current window. If the detector gives a negative response, the corresponding spectral coefficient is left without any correction.

This can be presented by the following formula:

$$\tilde{C}_{ij} = \begin{cases} \underset{m \times n}{MED}\left(C_{ij}\right) & \text{, if (2) holds} \\ C_{ij} & \text{, otherwise} \end{cases} \qquad (3)$$

where C_{ij} is the amplitude spectrum's coefficient at position (i, j) before the processing, $\tilde{C}_{ij}$ is the value of C_{ij} after the processing.

To choose a filter window, we have to take into account the following considerations. The larger windows (for example, 7×7 and higher) ensure much better detection of the peaks with a minimal level of the misdetections or without them at all, but at the same time the computing performance drops. So the choice of the filter window size is a choice of the equilibrium point between the performance and quality.

The example of the filtering using the filter (3) and the detector (2) is shown on Fig.2 (the filtering result of the Fig.2a is shown on Fig. 2g). One may compare this result to the commonly used technique (see previous section). The result of the filtering using the approach presented here doesn't contain any periodic diagonal lines. To convince, one can take a look to the result of Sobel edge detection (Fig. 2i) applied to the resulting image on Fig. 2g): no diagonal structure is detected.

The filter (3) is an excellent choice for the truly periodic noise, or noise, which is close to the truly periodic one. In other words, this filter is the best for the periodic noise that has singular peaks in spectrum, i.e. each peak consists of one or two spectral coefficients.

On the other hand, its results are not so fascinating for more complicate case of the quasi-periodic noise. To be able to remove this kind of noise we are going to generalize the filter (3) in the following way.

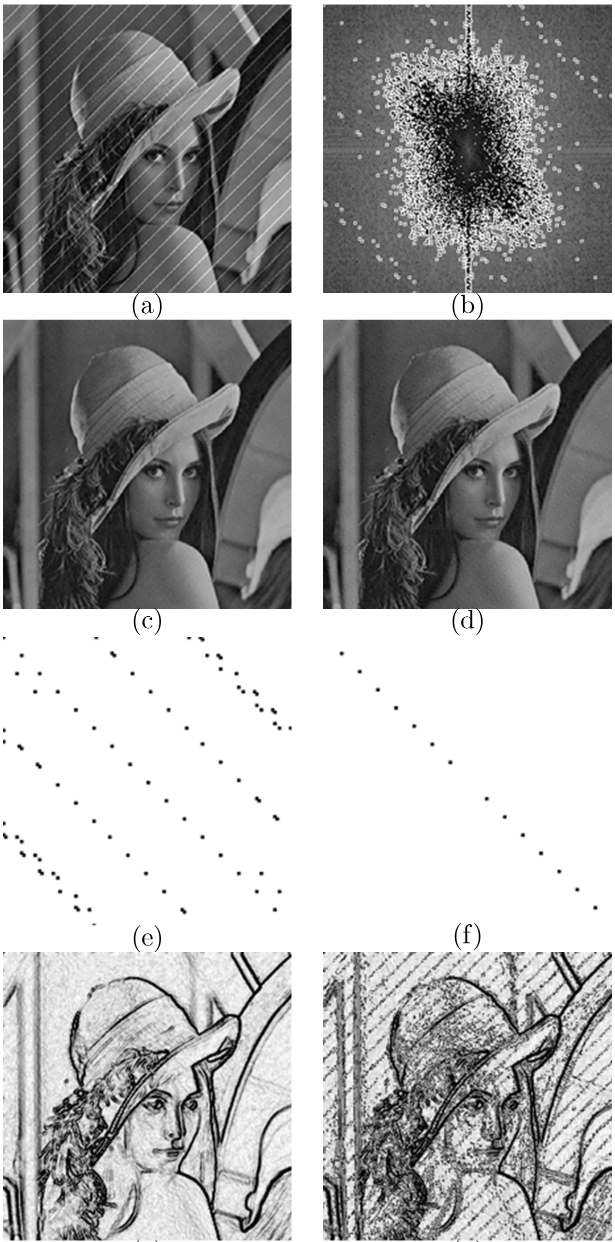

Figure 2: Filtering using mask obtained by thresholding and criterion (2). (a) Lena with periodic structure; (b) Amplitude spectrum of (a) with detected edges; (c) Image (a) filtered by spectral median filter (see section 3) with 7×7 filtering window, 5×5 frequency preservation window, 3 threshold; (d) Image (a) with peaks replaced by 0, peaks were extracted using mask (f); (e) Peaks detected by the median criterion (2); (f) Peaks detected by spectrum's thresholding with the threshold equal to 1. (g) Edge detection applied to (c), as you see, image is clear of periodic structure; (h) Edge detection applied to (d), here the periodic structure is still present.

4 GAUSSIAN MEDIAN FILTER

By observing a lot of images with a periodic noise, which is more complicate than truly horizontal or vertical stripes, or different quasi-periodic structures, it is visible that the peaks in the amplitude spectrum do not consist of one or two coefficients. They look more like a hill. It means that the replacement of the highest point of this hill by a local median decreases the intensity of the periodic noise, but does not remove it completely because the hill is still there, despite it is smaller. The Gaussian median filter, which will be described below, has a great advantage: it removes the whole hill. This leads to the dramatic improvement of the filtering result. It can be seen from the examples below.

This filter is also based on the peak detection scheme (2). The key point is that this filter corrects not only the coefficient of interest, but all the amplitude spectrum coefficients within the filter window. Mathematically it can be described in the following way:

$$\tilde{D}^{ij}_{mn} = \begin{cases} D^{ij}_{mn} \circ G_{mn} & \text{, if (2) holds} \\ D^{ij}_{mn} & \text{, otherwise} \end{cases} \quad (4)$$

where D^{ij}_{mn} is a $m \times n$ window around the (i, j)-th coefficient of the amplitude spectrum, which is passed to criterion (2). $\tilde{D}^{ij}_{mn}$ is the same window after the filtering. The "$\circ$" sign represents elementwise multiplication of two $m \times n$ matrixes. G_{mn} here is a $m \times n$ matrix, whose (x, y)-th element is defined by the formula:

$$G_{mn}(x, y) = 1 - Ae^{-B(x^2+y^2)} \quad (5)$$
$$x = 0, \ldots, n-1;\ y = 0, \ldots, m-1$$

where $0 < A \leq 1$ is the scaling coefficient along z axis and $0 < B \leq 1$ is a scaling coefficient along x and y axes.

The equation (5) defines a Gaussian-like surface. That's why this filter was called a Gaussian median filter.

Fig. 3 shows close-up of filtering: Fig. 3a represents a typical surface G, with $A = 1$, $B = 0.1$, Fig. 3b shows one of the peaks ($m = n = 11$) taken from the Clown image (Fig. 1c,d) spectrum and Fig. 3c shows the result of filtering of this peak by elementwise multiplication of 3a and 3b.

What is a role of the scaling coefficients A and B? By varying parameter B, the width of the hole of the Gaussian-like surface (Fig. 3) can be changed. The smaller values of B make the hole wider, which means that the surface can cover the whole hill and eliminate it. On the other hand it is clear that B can be taken so large that the surface will cover only peak itself (only central coefficient of the filter window), and thus this filter will perform similarly to the spectral median filter. Parameter A affects the amplitude of the surface. Generally it is better to keep it equal to 1, which means that

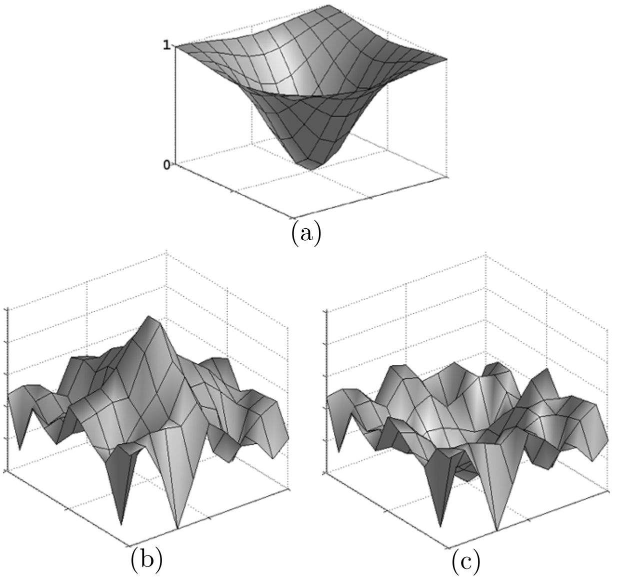

Figure 3: Gaussian Filter demonstration. (a) 11×11 Gaussian-like surface (5), $A = 1, B = 0.1$; (b) Peak from the spectrum (11×11 window); (c) Filtered peak (by elementwise multiplication of (a) and (b)) - as ypu can see, the whole peak disappeared.

peak will be completely removed with its central coefficient set to 0. Another way is to recalculate it for every filtering window in such a fashion that after the filtering the central coefficient in the window will become equal to the local median, instead of zero. But this will lead to significant complications of the algorithm from computational point of view, while the result will not differ too much.

5 SIMULATION RESULTS

Before going any further one more aspect of the processing must be considered. This aspect is the processing of the images whose sizes differ from the power of two. This is the limitation of the Cooley-Tukey's FFT algorithm. To overcome this limitation image can be simply padded with zeroes to have sizes of the closest power of two.

Figures 4, 5 show the comparison of filtering for three filtering techniques: spectrum thresholding, spectral median filter (3) and Gaussian median filter (4). Thresholding was done in the following way: first Fourier amplitude spectrum was calculated according to formula (1), then it was thresholded using some threshold. The threshold was chosen manually in such a fashion that the result will indicate as much peaks as possible. Then the result of thresholding was used to create a mask, by clearing the low frequencies and by placing in place of every peak a small circle (about 10 pixels wide) of zeros. All other coefficients in the mask were equal to one. The amplitude spectrum was then multiplied by the mask to remove the peaks [8]. The threshold for the proposed algorithms was chosen so, that the filter will only remove stripes without image distortion.

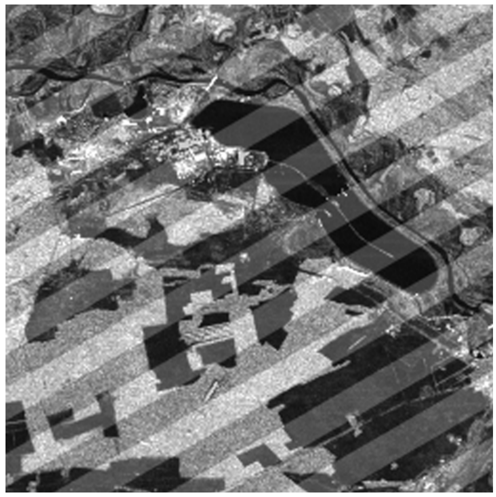

Figure 4: Satellite image with periodic stripes

As it is seen from example, best of all performed Gaussian median filter (Fig. 5a), median (5b) was also good, but because it filtered only one pixel in every window, it didn't remove the whole periodic pattern. The thresholding had left quite much of periodic structure (Fig. 5c,d), because it detected only the strongest peaks (Fig. 5f), while there are much more, as it can be seen from Fig. 5e.

6 CONCLUSIONS

A problem of the periodic and quasi-periodic noise filtering has been considered in this paper. This kind of noise implies the appearance of the peaks in image's amplitude spectrum. So the noise elimination is reduced to elimination of these peaks. The existing techniques of Wiener filtering and amplitude spectrum thresholding are not good enough. The Wiener filter is powerful and effective, but it is too complicate from the computing point of view and requires a precise noise model. The thresholding strongly depends on a choice of the threshold parameters and it is effective just for the pure periodic noise. So we developed a new effective filters with a semi-automatic peak detector.

Another goal, which was important for us, is a computing simplicity, for example in comparison to the Wiener filter. The designed spectral median and spectral Gaussian filters are almost as fast as the usual median filter, although one must not forget about the Fourier spectrum calculation. The most important advantage of the proposed solutions is their adaptation not only to pure periodic noise, but to different quasi-periodic structures, also as to the images whose sizes are not equal to power of two.

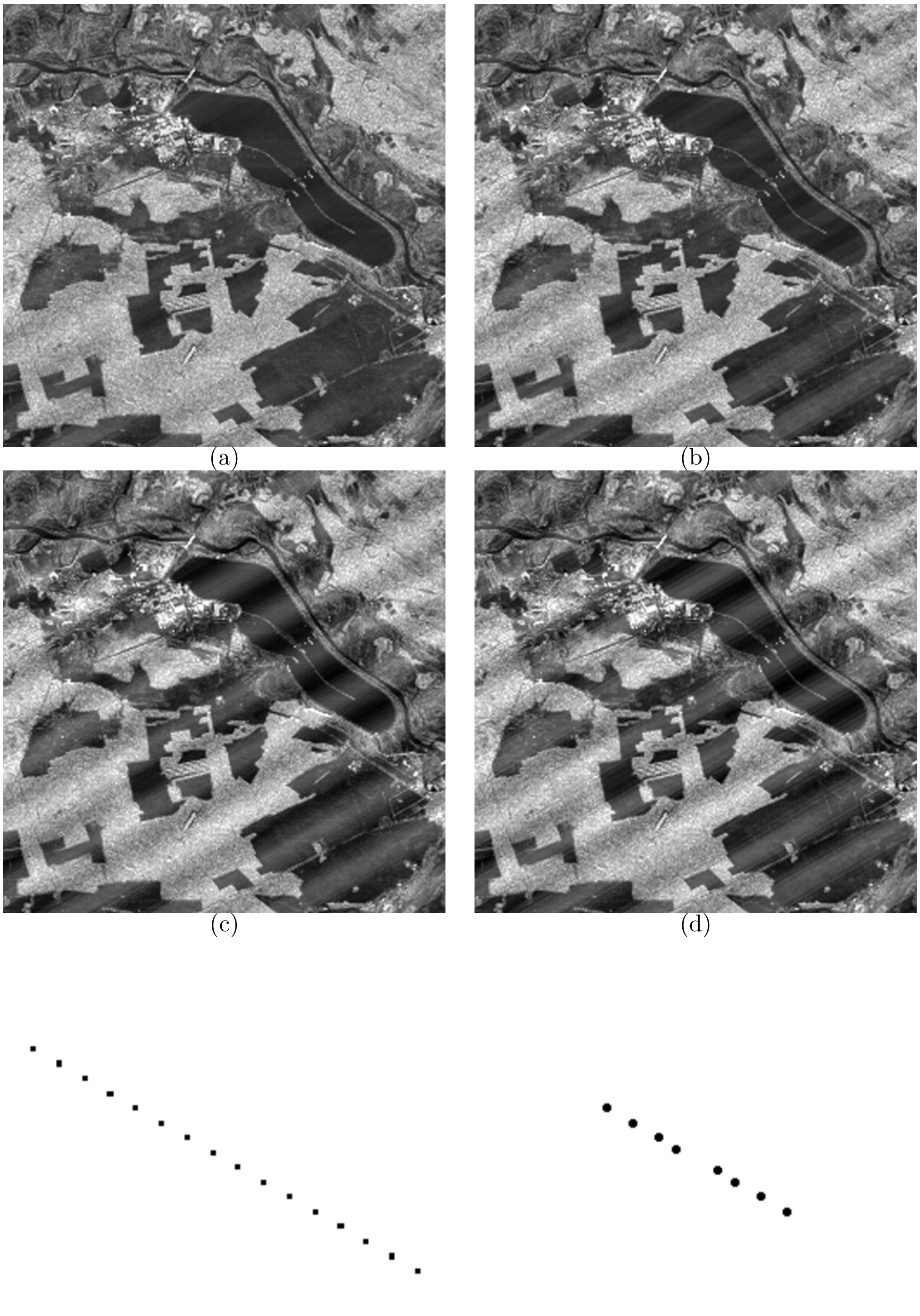

Figure 5: Filtering of Fig. 4. (a) Gaussian median filter (4) with 11×11 filteing and preservation windows, $\theta = 8, A = 1, B = 0.1$; (b) Spectral median filter (3) with 11×11 filteing and preservation windows, $\theta = 6$; (c) Spectrum thresholding, with threshold equal to 0.8; (d) Spectrum thresholding, with threshold equal to 1.7; (e) Coefficients that were filtered to obtain image (a); (f) Mask, used to obtain image (d).

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